FORMAL AND PHISYCAL ENVELOPS OF QUASI – HARMONIC OSCILLATIONS

Bogachev V.M., Solomatin D.A. kuleshov@srv-vmss.mpei.ac.ru

Moscow Power Engineering Institute (Technical University)

ABSTRACT

It is shown in this paper, that at analysis of frequency selective system response on the signal $v(t)=\mathrm{Re}(V \exp(j\omega_0+j\phi))$ it is useful to introduce concepts of "formal" and "physical" envelope of quasi-harmonic oscillation. Formal envelope does not depend on external phase ϕ, but it contains small components with frequencies close to $2\omega_0$. Physical envelope is obtained by informal transformation of standard solution, does not contain quickly oscillating terms, but it depends on external signal phase.

The exact solution of a starting operator equation of 2N-order can be presented in the form of sum of two operator N-order equations solutions. The first one determines the basic component of response. The second one defines the correction to the basic solution.

1. INTRODUCTION

The symbolic shortened equations method is offered by S.I. Yevtianov in 1946 as a variety of Van der Pol method - Ref. 1-3. Yevtianov`s method is widely used at investigation of linear and nonlinear frequency-selective systems due to the simple approximation of starting operator in vicinity of average band pass frequency and the simple algorithm of response component sorting by orders of smallness magnitudes. Later this method has been complementary based on spectral responses of frequency-selective networks and on the application of analytical signal theory - Ref. 4-7.

2. FORMAL AND PHYSICAL ENVELOPES

To simplify the treatment we will consider the band-pass filters (BPF) with arbitrary response characteristics (Butterworth, Chebyshev, Cauer etc.) as the model of a frequency-selective system.

The BPF amplitude-frequency response is even function of frequency $|K(j\cdot\omega)|=|K(-j\cdot\omega)|$ and has two pass band near frequencies ω_c and $-\omega_c$. The free frequencies is occurred as complex conjunction pairs $s_{-k} = s_k^*$ and focused in vicinity of central frequency $|s_k - j\omega_c| < \Delta\omega_c$. If free frequency is simple we can write

$$K(s)=\sum_{k=1}^{m}\left(\frac{A_k}{s-s_k}+\frac{A_{-k}}{s-s_{-k}}\right), \tag{1}$$

where A_k, A_{-k} is the residues of K(s) in pole of s_k with $A_{-k}=A_k^*$.According (1) K(s) can be conceived in form of sum of two frequency responses which formed K(s) in up and down part of complex plane $s=\sigma+j\omega$.

To understand the behavior of filter reaction on harmonic signal turn on $v(t)=\mathrm{Re}(V \exp(j\omega_0+j\phi))$ we find the images of input and output signals $V(s)=\mathrm{Re}(\exp(j\omega_0+j\phi)/(s-j\omega_0))$, $U(s)= V(s)\,(K_+(s) + K_-(s))$ and apply the second theorem of operator calculus. The time domain response is presented as sum of two components $u(t)= u_+(t) + u_-(t)$

$$u_+(t)=\mathrm{Re}\left[V\cdot e^{j\phi}\left(K_+(j\omega_0)e^{j\omega_0 t}+\sum_{k=1}^{m}B_k e^{s_k t}\right)\right] \tag{2}$$

$$u_-(t)=\mathrm{Re}\left[V\cdot e^{j\phi}\left(K_-(j\omega_0)e^{j\omega_0 t}+\sum_{k=1}^{m}B_{-k} e^{s_{-k} t}\right)\right] \tag{3}$$

where $B_k=A_k/(s_k - j\omega_0)$, $B_{-k} = A_{-k}/(s_{-k} - j\omega_0)$ residues of $K(s)/(s-j\omega_0)$.

The response on resonance action is mainly of interest, when the signal frequency is found in pass band $|\omega_k-\omega_0| \sim\Delta\omega_c$. In this case the frequency of all components is appropriate to introduce the complex envelope of response.

Once we have decided upon the reference frequency of external action ω_0 and to factor out $V \exp(j\omega_0 t+j\phi)$, we can rewrite (2),(3) as

$$u(t) = \mathrm{Re}(B_F(t)\, V \exp(j\omega_0 t+j\phi)) \tag{4}$$

In (4) we introduce the "formal" complex envelope of filter response by turn on the harmonic signal with V=1.

$$B_F(t)=K_+(j\omega_0)+\sum_{k=1}^{m}B_k e^{\sigma_k t+j(\omega_k-\omega_0)t}+ + K_-(j\omega_0)+\sum_{k=1}^{m}B_{-k} e^{\sigma_k t-j(\omega_k+\omega_0)t} \tag{5}$$

The preferences of this approach are that, first, the envelope $B_F(t)$ is introduced by simple standard procedure and, second, $B_F(t)$ is not depended from signal's phase. The disadvantages are that this function contains not only constant or slowly variable components, but also components $B_{-k}\cdot\exp(\sigma_k t -j(\omega_k+\omega_0)t)$, which oscillate with frequency $\omega_k+\omega_0$. This frequency is approximately twice the central frequency ω_c. To avoid the fast oscillations we exchange in (3) the component with negative frequency $B_{-k}\exp(s_{-k}\cdot t)$ by its conjunctives. It is evidence that this is not changed the solution. The time domain response may be written as:

$$u_-(t)=\mathrm{Re}\left[V\cdot K_-(j\omega_0)e^{j\omega_0 t+j\phi}+V\cdot\sum_{k=1}^{m}B_{-k}^{*} e^{s_{+k} t-j\phi}\right] \tag{6}$$

where $B_{-k}^* = A_k/(s_k +j\omega_0)$.

The sum response (2), (6) is consistent with complex envelope $B_p(t)$, which we named as "physical".

$$B_P(t)=K(j\omega_0)+\sum_{k=1}^{m}\left(B_k+B_{-k}^{*}e^{-j2\phi}\right)\cdot e^{\sigma_k t+j(\omega_k-\omega_0)t} \tag{7}$$

The physical envelope does not contain the fast oscillations, but as distinguish from formal envelope (5) this is depended on phase of input signal.

It is showed that on resonance action the component $u_-(t)$ at least have first order of smallness as compared with component of basic solution $u_+(t)$. Actually,

$$|K_-(j\omega_0)/K_+(j\omega_0)| < \delta/2,$$

$$|B_{-k}^*/B_k| = |(\sigma_k + j(\omega_k-\omega_0)/(\sigma_k + j(\omega_k+\omega_0)| \sim \delta.$$

It is taken into account that $(\omega_k-\omega_0) \sim \delta\omega_k$ and pole attenuation is sufficient small $2\sigma_k /\omega_k = \delta_k \sim \delta$. (where δ – the small parameter). By saving in full solution only basic components we obtain:

$$u_0(t) = Re(B_0(t)\ V \exp(j\omega_0 t+j\phi)).,$$

$$B_0(t)=B_{p+}(t)= B_{F+}(t). \qquad (8)$$

The correction to physical complex envelope of basic solution can be written in form

$$\Delta B(t) = K_-(j\omega_0) + \sum_{k=1}^{m} B_{-k}^* \cdot e^{\sigma_k t + j(\omega_k-\omega_0)t - j2\phi} \qquad (9)$$

and slowly depend on ω_0 when $\omega_0 \sim \omega_c$. In this case

$$B_{-k}^* = (A_k/(\sigma_k+j(\omega_k+\omega_0) \approx -jA_k/(\omega_k+\omega_0) \qquad (10)$$

and expression for correction $\Delta B(t)$ can be simplified

$$\Delta B(t) = K_-(j\omega_0) - j\frac{dB_0}{dt} \cdot \frac{e^{-j2\phi}}{\omega_c + \omega_0} \qquad (11)$$

3. SHORTENED OPERATIONAL EQUATIONS

Previously we consider transient responses of BPF when one turns on the jump of harmonic voltage with constant frequency and given initial phase. Now we obtain the differential equations, which defines in operator form the complex envelope of quasi-harmonic signal. It is examine the action on system by quasi-harmonic signal with given time-depended complex envelope $\overline{V}(t) = V(t)\exp(j\phi(t))$:

$$v(t) = Re\left(\overline{V}(t)\exp(j\omega_0 t)\right)$$

The components of response can be represented by complex form. For example u_0 can be written as

$$u_0(t) = Re\left(\overline{U}(t)\exp(j\omega_0 t)\right)$$

where $\overline{U}(t) = U(t)\exp(j\phi_0(t))$ – the complex envelope.

It can be changed $K_+(s)$ by rational function $K_+(s) = P(s)/Q(s)$, where P(s) and Q(s) is a polynomial with complex coefficients, we can write the differential equation for time domain response in form

$$Q(s)\cdot u_0(t) = P(s)\cdot v(t), \quad s = d/dt. \qquad (12)$$

Going to complex envelope in (12) and omitting the sight of Re in left and right part of equation we obtain

$$Q(s)\overline{U}(t)\exp(j\omega_0 t) = P(s)\overline{V}(t)\exp(j\omega_0 t) \qquad (13)$$

When we apply, according -Ref.1-3 the shifting theorem of operator calculus to operator's polynomial, in place of (13) we have:

$$Q(s + j\omega_0)\overline{U}(t) = P(s + j\omega_0)\overline{V}(t) \qquad (14)$$

or

$$\overline{U}(t) = K_+(s + j\omega_0)\overline{V}(t) \qquad (15).$$

Note, that (14) and (15) are the exact equations for complex envelope $\overline{U}(t)$. They had been obtained from (12) by the simple exchange of variable. This equation coincides with analogous Yevtianov symbolic equations within of second order of smallness magnitudes. The distinguish is explained by different approach to approximation of frequency response for K(s) near pass band.

The equation for complex envelope of correction $\Delta u(t)$ to basic equation can be found from (9),(11). In particular by resonance action, we have according (11)

$$\Delta\overline{U}(t) = K_-(j\omega_0)\overline{V}(t) + \frac{sK_+(s + j\omega_0)}{j(\omega_c + \omega_0)}\overline{V}(t)^* \qquad (16)$$

Equation for complex envelope can be solved on base of Laplace transform (if complex amplitude of input signal has simple image) or with help of Duhamel integral. The last presentation is applied for complex time-depended functions.

4. CONCLUSION

In this paper, the modified proof of the method of shortened operator equations was obtained. In this approach the prerequisite to slowly change of complex envelope of response did not used, in contrast to Van der Pole method, Yevtianov method and other similar approaches -Ref.1-7. This condition including the estimation of accuracy of first order approximation expands the region of application of this method. As examples the transients in BPF with Batterworth, Chebyshev and Cauer filters were examined.

5. REFERENCES

1. Yevtianov S.I. Transients in receivers and amplifiers (in Russian). Moscow, Sviazizdat, 1948. - 210 p.

2. Kapranov M.V., Kuleshov V.N., Utkin G.M. Oscillation theory in radio engineering (in Russian). Moscow, Nauka, 1984. - 320 p.

3. Bogachev V.M., Lysenko V.G., Cmolskiy S.M. Transistor oscillators and autodynes (in Russian). Moscow, Moscow Power Engineering Institute, 1993. - 344 p.

4. Radio receivers. Editor V.I. Siforov (in Russian). Moscow, Sov. radio, 1974. - 560 p.

5. Vainstein V.A., Vakman D.E. Frequency separation in oscillation and wave theory (in Russian). Moscow, Nauka, 1972. - 288 p.

6. Baskakov S.I. Radio technical networks and signals (in Russian). Moscow, Vyschaia shkola, 1983. - 536 p. .

7. Applied mathematical methods in radio engineering. Editor G.V. Obrezkov (in Russian). Moscow, Vyschaia shkola, 1985. - 343 p.